# Mathematica 11.3 Integration Test Results

## Test results for the 42 problems in "1.2.2.7 P(x) (d+e x^2)^q (a+b x^2+c x^4)^p.m"

### Problem 1: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)^3}{\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 453 leaves, 15 steps):

$$\frac{e \left(21 B c d^2+21 A c d e-5 a B e^2\right) x \sqrt{a+c x^4}}{21 c^2}+\frac{e^2 \left(3 B d+A e\right) x^3 \sqrt{a+c x^4}}{5 c}+\frac{B e^3 x^5 \sqrt{a+c x^4}}{7 c}+\frac{\left(5 B c d^3+15 A c d^2 e-9 a B d e^2-3 a A e^3\right) x \sqrt{a+c x^4}}{5 c^{3/2} \left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{1}{5 c^{7/4} \sqrt{a+c x^4}} a^{1/4} \left(5 B c d^3+15 A c d^2 e-9 a B d e^2-3 a A e^3\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]+\left(\left(105 A c^2 d^3+25 a^2 B e^3-105 a c d e \left(B d+A e\right)-63 a^{3/2} \sqrt{c}\, e^2 \left(3 B d+A e\right)+105 \sqrt{a}\, c^{3/2} d^2 \left(B d+3 A e\right)\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(210 a^{1/4} c^{9/4} \sqrt{a+c x^4}\right)$$

Result (type 4, 323 leaves):

$$\frac{1}{105\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,c^2\sqrt{a+c\,x^4}}\left(-\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,e\,x\,\left(a+c\,x^4\right)\,\left(25\,a\,B\,e^2-21\,A\,c\,e\,\left(5\,d+e\,x^2\right)-3\,B\,c\,\left(35\,d^2+21\,d\,e\,x^2+5\,e^2\,x^4\right)\right)-21\,\sqrt{a}\,\sqrt{c}\,\left(-5\,B\,c\,d^3-15\,A\,c\,d^2\,e+9\,a\,B\,d\,e^2+3\,a\,A\,e^3\right)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\left(-105\,i\,A\,c^2\,d^3-25\,i\,a^2\,B\,e^3+105\,i\,a\,c\,d\,e\,\left(B\,d+A\,e\right)+63\,a^{3/2}\sqrt{c}\,e^2\,\left(3\,B\,d+A\,e\right)-105\,\sqrt{a}\,c^{3/2}\,d^2\,\left(B\,d+3\,A\,e\right)\right)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

## Problem 2: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)^2}{\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 367 leaves, 12 steps):

$$\frac{e\,\left(2\,B\,d+A\,e\right)\,x\,\sqrt{a+c\,x^4}}{3\,c}+\frac{B\,e^2\,x^3\,\sqrt{a+c\,x^4}}{5\,c}+\frac{\left(5\,B\,c\,d^2+10\,A\,c\,d\,e-3\,a\,B\,e^2\right)\,x\,\sqrt{a+c\,x^4}}{5\,c^{3/2}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{1}{5\,c^{7/4}\sqrt{a+c\,x^4}}a^{1/4}\,\left(5\,B\,c\,d^2+10\,A\,c\,d\,e-3\,a\,B\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\left(\left(15\,A\,c^{3/2}\,d^2-9\,a^{3/2}\,B\,e^2-5\,a\,\sqrt{c}\,e\,\left(2\,B\,d+A\,e\right)+15\,\sqrt{a}\,c\,d\,\left(B\,d+2\,A\,e\right)\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(30\,a^{1/4}\,c^{7/4}\,\sqrt{a+c\,x^4}\right)$$

Result (type 4, 260 leaves):

$$\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ \sqrt{c}\ e\,x\,\left(10\,B\,d+5\,A\,e+3\,B\,e\,x^2\right)\,\left(a+c\,x^4\right)-\right.$$
$$3\,\sqrt{a}\,\left(-5\,B\,c\,d^2-10\,A\,c\,d\,e+3\,a\,B\,e^2\right)\,\sqrt{1+\frac{c\,x^4}{a}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]+$$
$$\left(-15\,i\,A\,c^{3/2}\,d^2+9\,a^{3/2}\,B\,e^2+5\,i\,a\,\sqrt{c}\ e\,\left(2\,B\,d+A\,e\right)-15\,\sqrt{a}\ c\,d\,\left(B\,d+2\,A\,e\right)\right)$$
$$\left.\sqrt{1+\frac{c\,x^4}{a}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]\right)\Bigg/\left(15\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ c^{3/2}\,\sqrt{a+c\,x^4}\right)$$

## Problem 3: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)}{\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 277 leaves, 8 steps):

$$\frac{B\,e\,x\,\sqrt{a+c\,x^4}}{3\,c}+\frac{\left(B\,d+A\,e\right)\,x\,\sqrt{a+c\,x^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{1}{c^{3/4}\,\sqrt{a+c\,x^4}}$$
$$a^{1/4}\,\left(B\,d+A\,e\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+$$
$$\frac{1}{6\,c^{5/4}\,\sqrt{a+c\,x^4}}a^{1/4}\,\left(3\,\sqrt{c}\,\left(B\,d+A\,e\right)+\frac{3\,A\,c\,d-a\,B\,e}{\sqrt{a}}\right)$$
$$\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 202 leaves):

$$\left(B\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ e\,x\,\left(a+c\,x^4\right)+3\,\sqrt{a}\,\sqrt{c}\,\left(B\,d+A\,e\right)\,\sqrt{1+\frac{c\,x^4}{a}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]+\right.$$
$$\left.i\,\left(-3\,A\,c\,d+a\,B\,e+3\,i\,\sqrt{a}\,\sqrt{c}\,\left(B\,d+A\,e\right)\right)\,\sqrt{1+\frac{c\,x^4}{a}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]\right)\Bigg/$$
$$\left(3\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ c\,\sqrt{a+c\,x^4}\right)$$

## Problem 4: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{\sqrt{a + c x^4}} \, dx$$

Optimal (type 4, 226 leaves, 3 steps):

$$\frac{B x \sqrt{a + c x^4}}{\sqrt{c} \left(\sqrt{a} + \sqrt{c} x^2\right)} - \frac{a^{1/4} B \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{c^{3/4} \sqrt{a + c x^4}} + \frac{1}{2 c^{3/4} \sqrt{a + c x^4}} a^{1/4} \left(B + \frac{A \sqrt{c}}{\sqrt{a}}\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 131 leaves):

$$\left(\sqrt{1 + \frac{c x^4}{a}} \left(\sqrt{a} B \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \left(\sqrt{a} B + i A \sqrt{c}\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]\right)\right) \Bigg/ \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{c} \sqrt{a + c x^4}\right)$$

## Problem 5: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{\left(d + e x^2\right) \sqrt{a + c x^4}} \, dx$$

Optimal (type 4, 369 leaves, 3 steps):

$$-\frac{(B d-A e)\,\text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\;x}{\sqrt{d}\;\sqrt{e}\;\sqrt{a+c x^4}}\right]}{2\sqrt{d}\;\sqrt{e}\;\sqrt{c d^2+a e^2}}-$$

$$\left(\left(\sqrt{a}\;B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2\,a^{1/4}c^{1/4}\left(\sqrt{c}\;d-\sqrt{a}\;e\right)\sqrt{a+c x^4}\right)+$$

$$\left(a^{3/4}\left(\frac{\sqrt{c}\;d}{\sqrt{a}}+e\right)^2(B d-A e)\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\;d-\sqrt{a}\;e\right)^2}{4\sqrt{a}\;\sqrt{c}\;d e},2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4\,c^{1/4}d e\left(c d^2-a e^2\right)\sqrt{a+c x^4}\right)$$

Result (type 4, 138 leaves):

$$-\left(\left(\mathrm{i}\sqrt{1+\frac{c x^4}{a}}\left(B d\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]+(-B d+A e)\right.\right.\right.$$

$$\left.\left.\left.\text{EllipticPi}\left[-\frac{\mathrm{i}\sqrt{a}\;e}{\sqrt{c}\;d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]\right)\right)\Big/\left(\sqrt{\frac{\mathrm{i}\sqrt{c}}{\sqrt{a}}}\;d e\sqrt{a+c x^4}\right)\right)$$

## Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^2}{\left(d+e x^2\right)^2\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 641 leaves, 6 steps):

$$\frac{\sqrt{c}\,(B d-A e)\,x\sqrt{a+c x^4}}{2 d\,(c d^2+a e^2)\,(\sqrt{a}+\sqrt{c}\,x^2)}-\frac{e\,(B d-A e)\,x\sqrt{a+c x^4}}{2 d\,(c d^2+a e^2)\,(d+e x^2)}-$$

$$\frac{(B c d^3-3 A c d^2 e-a B d e^2-a A e^3)\,\text{ArcTan}\Big[\frac{\sqrt{c d^2+a e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c x^4}}\Big]}{4 d^{3/2}\sqrt{e}\,(c d^2+a e^2)^{3/2}}-$$

$$\left(a^{1/4} c^{1/4}\,(B d-A e)\,(\sqrt{a}+\sqrt{c}\,x^2)\sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\;\text{EllipticE}\Big[2\,\text{ArcTan}\Big[\frac{c^{1/4} x}{a^{1/4}}\Big],\frac{1}{2}\Big]\right)\Big/$$

$$\left(2 d\,(c d^2+a e^2)\sqrt{a+c x^4}\right)+$$

$$\frac{A c^{1/4}\,(\sqrt{a}+\sqrt{c}\,x^2)\sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\;\text{EllipticF}\Big[2\,\text{ArcTan}\Big[\frac{c^{1/4} x}{a^{1/4}}\Big],\frac{1}{2}\Big]}{2 a^{1/4} d\,(\sqrt{c}\,d-\sqrt{a}\,e)\sqrt{a+c x^4}}+$$

$$\Bigg((\sqrt{c}\,d+\sqrt{a}\,e)\,(B c d^3-3 A c d^2 e-a B d e^2-a A e^3)\,(\sqrt{a}+\sqrt{c}\,x^2)$$

$$\sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\;\text{EllipticPi}\Big[-\frac{(\sqrt{c}\,d-\sqrt{a}\,e)^2}{4\sqrt{a}\,\sqrt{c}\,d e},2\,\text{ArcTan}\Big[\frac{c^{1/4} x}{a^{1/4}}\Big],\frac{1}{2}\Big]\Bigg)\Big/$$

$$\left(8 a^{1/4} c^{1/4} d^2 e\,(\sqrt{c}\,d-\sqrt{a}\,e)\,(c d^2+a e^2)\sqrt{a+c x^4}\right)$$

Result (type 4, 297 leaves) :

$$\frac{1}{2 d^2\sqrt{a+c x^4}}\Bigg(\frac{d e\,(-B d+A e)\,x\,(a+c x^4)}{(c d^2+a e^2)\,(d+e x^2)}-$$

$$\Bigg(i\sqrt{1+\frac{c x^4}{a}}\Bigg(i\sqrt{a}\,\sqrt{c}\,d e\,(B d-A e)\,\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\Big],-1\Big]+$$

$$\sqrt{c}\,d\,(\sqrt{c}\,d-i\sqrt{a}\,e)\,(B d-A e)\,\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\Big],-1\Big]+$$

$$(-B c d^3+3 A c d^2 e+a B d e^2+a A e^3)$$

$$\text{EllipticPi}\Big[-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},i\,\text{ArcSinh}\Big[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\Big],-1\Big]\Bigg)\Bigg)\Big/\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,(c d^2 e+a e^3)\right)\Bigg)$$

## Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{\left(d + e x^2\right)^3 \sqrt{a + c x^4}} \, dx$$

Optimal (type 4, 875 leaves, 7 steps):

$$\frac{\sqrt{c} \left(5 B c d^3 - 9 A c d^2 e - a B d e^2 - 3 a A e^3\right) x \sqrt{a + c x^4}}{8 d^2 \left(c d^2 + a e^2\right)^2 \left(\sqrt{a} + \sqrt{c} x^2\right)} - \frac{e \left(B d - A e\right) x \sqrt{a + c x^4}}{4 d \left(c d^2 + a e^2\right) \left(d + e x^2\right)^2} - \frac{e \left(5 B c d^3 - 9 A c d^2 e - a B d e^2 - 3 a A e^3\right) x \sqrt{a + c x^4}}{8 d^2 \left(c d^2 + a e^2\right)^2 \left(d + e x^2\right)} + \left(\left(3 A e \left(5 c^2 d^4 + 2 a c d^2 e^2 + a^2 e^4\right) - B \left(3 c^2 d^5 - 10 a c d^3 e^2 - a^2 d e^4\right)\right) \text{ArcTan}\left[\frac{\sqrt{c d^2 + a e^2} x}{\sqrt{d} \sqrt{e} \sqrt{a + c x^4}}\right]\right) \Big/ \left(16 d^{5/2} \sqrt{e} \left(c d^2 + a e^2\right)^{5/2}\right) - \left(a^{1/4} c^{1/4} \left(5 B c d^3 - 9 A c d^2 e - a B d e^2 - 3 a A e^3\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(8 d^2 \left(c d^2 + a e^2\right)^2 \sqrt{a + c x^4}\right) + \left(c^{1/4} \left(4 A c d^2 + \sqrt{a} \sqrt{c} d \left(B d - A e\right) + a e \left(B d + 3 A e\right)\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(8 a^{1/4} d^2 \left(\sqrt{c} d - \sqrt{a} e\right) \left(c d^2 + a e^2\right) \sqrt{a + c x^4}\right) - \left(\left(\sqrt{c} d + \sqrt{a} e\right) \left(3 A e \left(5 c^2 d^4 + 2 a c d^2 e^2 + a^2 e^4\right) - B \left(3 c^2 d^5 - 10 a c d^3 e^2 - a^2 d e^4\right)\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticPi}\left[-\frac{\left(\sqrt{c} d - \sqrt{a} e\right)^2}{4 \sqrt{a} \sqrt{c} d e}, 2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(32 a^{1/4} c^{1/4} d^3 e \left(\sqrt{c} d - \sqrt{a} e\right) \left(c d^2 + a e^2\right)^2 \sqrt{a + c x^4}\right)$$

Result (type 4, 453 leaves):

$$\frac{1}{8 d^3 e \left(c d^2+a e^2\right)^2 \sqrt{a+c x^4}}\left(-\frac{1}{\left(d+e x^2\right)^2} d e^2 x \left(a+c x^4\right) \left(2 d (B d-A e) \left(c d^2+a e^2\right)+\left(5 B c d^3-9 A c d^2 e-a B d e^2-3 a A e^3\right) \left(d+e x^2\right)\right)-\frac{1}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} i \sqrt{1+\frac{c x^4}{a}} \left(-i \sqrt{a} \sqrt{c}\, d e \left(-5 B c d^3+9 A c d^2 e+a B d e^2+3 a A e^3\right) \text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+\sqrt{c}\, d \left(\sqrt{c}\, d-i \sqrt{a}\, e\right) \left(A e \left(-7 c d^2+2 i \sqrt{a} \sqrt{c}\, d e-3 a e^2\right)+B d \left(3 c d^2-2 i \sqrt{a} \sqrt{c}\, d e-a e^2\right)\right) \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+\left(3 A e \left(5 c^2 d^4+2 a c d^2 e^2+a^2 e^4\right)+B \left(-3 c^2 d^5+10 a c d^3 e^2+a^2 d e^4\right)\right) \text{EllipticPi}\left[-\frac{i \sqrt{a}\, e}{\sqrt{c}\, d}, i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)\right)$$

## Problem 8: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)^3}{\left(a+c x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 912 leaves, 12 steps) :

$$\frac{1}{2\,a\,c^2\,\sqrt{a+c\,x^4}}\,x\left(A\,c\,d\left(c\,d^2-3\,a\,e^2\right)-a\,B\,e\left(3\,c\,d^2-a\,e^2\right)+c\left(B\,c\,d^3+3\,A\,c\,d^2\,e-3\,a\,B\,d\,e^2-a\,A\,e^3\right)x^2\right)+$$

$$\frac{B\,e^3\,x\,\sqrt{a+c\,x^4}}{3\,c^2}+\frac{e^2\left(3\,B\,d+A\,e\right)x\,\sqrt{a+c\,x^4}}{c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-$$

$$\frac{\left(B\,c\,d^3+3\,A\,c\,d^2\,e-3\,a\,B\,d\,e^2-a\,A\,e^3\right)x\,\sqrt{a+c\,x^4}}{2\,a\,c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{1}{c^{7/4}\,\sqrt{a+c\,x^4}}$$

$$a^{1/4}\,e^2\left(3\,B\,d+A\,e\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+$$

$$\left(\left(B\,c\,d^3+3\,A\,c\,d^2\,e-3\,a\,B\,d\,e^2-a\,A\,e^3\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2\,a^{3/4}\,c^{7/4}\,\sqrt{a+c\,x^4}\right)-\frac{1}{6\,c^{9/4}\,\sqrt{a+c\,x^4}}$$

$$a^{3/4}\,B\,e^3\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{2\,c^{7/4}\,\sqrt{a+c\,x^4}}$$

$$a^{1/4}\,e^2\left(3\,B\,d+A\,e\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+$$

$$\left(e\left(3\,B\,c\,d^2+3\,A\,c\,d\,e-a\,B\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2\,a^{1/4}\,c^{9/4}\,\sqrt{a+c\,x^4}\right)+$$

$$\left(\left(A\,c^2\,d^3+a^2\,B\,e^3-3\,a\,c\,d\,e\left(B\,d+A\,e\right)+a^{3/2}\,\sqrt{c}\,e^2\left(3\,B\,d+A\,e\right)-\sqrt{a}\,c^{3/2}\,d^2\left(B\,d+3\,A\,e\right)\right)\right.$$

$$\left.\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4\,a^{5/4}\,c^{9/4}\,\sqrt{a+c\,x^4}\right)$$

Result (type 4, 351 leaves) :

$$\frac{1}{6\,a\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,c^2\,\sqrt{a+c\,x^4}}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\,\left(3\,A\,c\,\left(-a\,e^2\,\left(3\,d+e\,x^2\right)+c\,d^2\,\left(d+3\,e\,x^2\right)\right)+\right.\right.$$
$$\left.B\,\left(5\,a^2\,e^3+3\,c^2\,d^3\,x^2+a\,c\,e\,\left(-9\,d^2-9\,d\,e\,x^2+2\,e^2\,x^4\right)\right)\right)+3\,\sqrt{a}\,\sqrt{c}$$
$$\left(-B\,c\,d^3-3\,A\,c\,d^2\,e+9\,a\,B\,d\,e^2+3\,a\,A\,e^3\right)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+$$
$$\left(-3\,i\,A\,c^2\,d^3+5\,i\,a^2\,B\,e^3-9\,i\,a\,c\,d\,e\,\left(B\,d+A\,e\right)-9\,a^{3/2}\,\sqrt{c}\,e^2\,\left(3\,B\,d+A\,e\right)+\right.$$
$$\left.\left.3\,\sqrt{a}\,c^{3/2}\,d^2\,\left(B\,d+3\,A\,e\right)\right)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

## Problem 9: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)^2}{\left(a+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 694 leaves, 10 steps):

$$\frac{x\left(A c d^2-2 a B d e-a A e^2+\left(B c d^2+2 A c d e-a B e^2\right) x^2\right)}{2 a c \sqrt{a+c x^4}}+$$

$$\frac{B e^2 x \sqrt{a+c x^4}}{c^{3/2}\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{\left(B c d^2+2 A c d e-a B e^2\right) x \sqrt{a+c x^4}}{2 a c^{3/2}\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{1}{c^{7/4}\sqrt{a+c x^4}}$$

$$a^{1/4} B e^2\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\; \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+$$

$$\left(\left(B c d^2+2 A c d e-a B e^2\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\; \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2 a^{3/4} c^{7/4}\sqrt{a+c x^4}\right)+\frac{1}{2 c^{7/4}\sqrt{a+c x^4}}$$

$$a^{1/4} B e^2\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\; \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+$$

$$\left(e\left(2 B d+A e\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\; \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2 a^{1/4} c^{5/4}\sqrt{a+c x^4}\right)-\left(\left(B c d^2+2 A c d e-a B e^2-\frac{\sqrt{c}\left(A c d^2-2 a B d e-a A e^2\right)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\right.$$

$$\left.\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\; \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4 a^{3/4} c^{7/4}\sqrt{a+c x^4}\right)$$

Result (type 4, 282 leaves) :

$$\left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\;\sqrt{c}\; x\left(-a A e^2+B c d^2 x^2-a B e\left(2 d+e x^2\right)+A c d\left(d+2 e x^2\right)\right)+\right.$$

$$\sqrt{a}\left(-B c d^2-2 A c d e+3 a B e^2\right)\sqrt{1+\frac{c x^4}{a}}\; \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\; x\right],-1\right]-$$

$$\left(i A c^{3/2} d^2+3 a^{3/2} B e^2+i a \sqrt{c}\; e\left(2 B d+A e\right)-\sqrt{a}\; c d\left(B d+2 A e\right)\right)\sqrt{1+\frac{c x^4}{a}}$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\; x\right],-1\right]\right)\Big/\left(2 a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\; c^{3/2}\sqrt{a+c x^4}\right)$$

# Problem 10: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)}{\left(a+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 395 leaves, 7 steps):

$$\frac{x \left(A c d-a B e+c (B d+A e) x^2\right)}{2 a c \sqrt{a+c x^4}}-\frac{(B d+A e) x \sqrt{a+c x^4}}{2 a \sqrt{c} \left(\sqrt{a}+\sqrt{c} x^2\right)}+\left((B d+A e) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(2 a^{3/4} c^{3/4} \sqrt{a+c x^4}\right)+\frac{B e \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{1/4} c^{5/4} \sqrt{a+c x^4}}+\left(\left(A c d-a B e-\sqrt{a} \sqrt{c} (B d+A e)\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(4 a^{5/4} c^{5/4} \sqrt{a+c x^4}\right)$$

Result (type 4, 218 leaves):

$$\left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \left(-a B e+B c d x^2+A c \left(d+e x^2\right)\right)-\sqrt{a} \sqrt{c} (B d+A e) \sqrt{1+\frac{c x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]+\left(-i A c d-i a B e+\sqrt{a} \sqrt{c} (B d+A e)\right) \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]\right) \Big/ \left(2 a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c \sqrt{a+c x^4}\right)$$

# Problem 11: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{\left(a+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 262 leaves, 4 steps):

$$\frac{x\left(A+B x^2\right)}{2 a \sqrt{a+c x^4}}-\frac{B x \sqrt{a+c x^4}}{2 a \sqrt{c}\left(\sqrt{a}+\sqrt{c}\, x^2\right)}+\frac{B\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} c^{3/4} \sqrt{a+c x^4}}-\left(\left(\sqrt{a}\, B-A \sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(4 a^{5/4} c^{3/4} \sqrt{a+c x^4}\right)$$

Result (type 4, 182 leaves):

$$\left(i \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c}\, x\left(A+B x^2\right)-\sqrt{a}\, B \sqrt{1+\frac{c x^4}{a}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+\left(\sqrt{a}\, B-i A \sqrt{c}\right) \sqrt{1+\frac{c x^4}{a}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)\right) \Big/ \left(2 a^{3/2}\left(\frac{i \sqrt{c}}{\sqrt{a}}\right)^{3/2} \sqrt{a+c x^4}\right)$$

## Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{\left(d+e x^2\right)\left(a+c x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 732 leaves, 9 steps):

$$\frac{x\left(A c d+a B e+c (B d-A e) x^2\right)}{2 a \left(c d^2+a e^2\right) \sqrt{a+c x^4}}-\frac{\sqrt{c} (B d-A e) x \sqrt{a+c x^4}}{2 a \left(c d^2+a e^2\right)\left(\sqrt{a}+\sqrt{c} x^2\right)}-\frac{e^{3/2} (B d-A e) \text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2} x}{\sqrt{d} \sqrt{e} \sqrt{a+c x^4}}\right]}{2 \sqrt{d} \left(c d^2+a e^2\right)^{3/2}}+$$

$$\left(c^{1/4} (B d-A e)\left(\sqrt{a}+\sqrt{c} x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2 a^{3/4}\left(c d^2+a e^2\right)\sqrt{a+c x^4}\right)-$$

$$\left(c^{1/4} e (B d-A e)\left(\sqrt{a}+\sqrt{c} x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2 a^{1/4}\left(\sqrt{c} d-\sqrt{a} e\right)\left(c d^2+a e^2\right)\sqrt{a+c x^4}\right)+$$

$$\left(\left(A c d+a B e-\sqrt{a} \sqrt{c} (B d-A e)\right)\left(\sqrt{a}+\sqrt{c} x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4 a^{5/4} c^{1/4}\left(c d^2+a e^2\right)\sqrt{a+c x^4}\right)+$$

$$\left(a^{3/4} e\left(\frac{\sqrt{c} d}{\sqrt{a}}+e\right)^2 (B d-A e)\left(\sqrt{a}+\sqrt{c} x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c} d-\sqrt{a} e\right)^2}{4 \sqrt{a} \sqrt{c} d e},2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4 c^{1/4} d\left(c^2 d^4-a^2 e^4\right)\sqrt{a+c x^4}\right)$$

Result (type 4, 432 leaves) :

$$\frac{1}{2\,a\,\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,d\,\left(c\,d^2+a\,e^2\right)\,\sqrt{a+c\,x^4}}$$

$$\left(A\,\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,c\,d^2\,x+a\,B\,\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,d\,e\,x+B\,\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,c\,d^2\,x^3-A\,\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,c\,d\,e\,x^3-\right.$$

$$\sqrt{a}\,\sqrt{c}\,d\,(B\,d-A\,e)\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+$$

$$\left(\sqrt{a}\,B-i\,A\,\sqrt{c}\right)\,d\,\left(\sqrt{c}\,d-i\,\sqrt{a}\,e\right)\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+$$

$$2\,i\,a\,B\,d\,e\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{i\,\sqrt{a}\,e}{\sqrt{c}\,d},\,i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.2\,i\,a\,A\,e^2\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{i\,\sqrt{a}\,e}{\sqrt{c}\,d},\,i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

## Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{\left(d+e\,x^2\right)^2\,\left(a+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 1494 leaves, 15 steps):

$$\frac{c\,x\,\left(A\,c\,d^2+2\,a\,B\,d\,e-a\,A\,e^2+\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)\,x^2\right)}{2\,a\,\left(c\,d^2+a\,e^2\right)^2\,\sqrt{a+c\,x^4}}+$$

$$\frac{\sqrt{c}\,e^2\,(B\,d-A\,e)\,x\,\sqrt{a+c\,x^4}}{2\,d\,\left(c\,d^2+a\,e^2\right)^2\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt{c}\,\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)\,x\,\sqrt{a+c\,x^4}}{2\,a\,\left(c\,d^2+a\,e^2\right)^2\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-$$

$$\frac{e^3\,(B\,d-A\,e)\,x\,\sqrt{a+c\,x^4}}{2\,d\,\left(c\,d^2+a\,e^2\right)^2\,\left(d+e\,x^2\right)}-\frac{e^{3/2}\,(B\,d-A\,e)\,\left(3\,c\,d^2+a\,e^2\right)\,\text{ArcTan}\left[\frac{\sqrt{c\,d^2+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c\,x^4}}\right]}{4\,d^{3/2}\,\left(c\,d^2+a\,e^2\right)^{5/2}}-$$

$$\frac{e^{3/2}\,\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)\,\text{ArcTan}\left[\frac{\sqrt{c\,d^2+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c\,x^4}}\right]}{2\,\sqrt{d}\,\left(c\,d^2+a\,e^2\right)^{5/2}}-$$

$$\left(a^{1/4}\,c^{1/4}\,e^2\,(B\,d-A\,e)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2\,d\,\left(c\,d^2+a\,e^2\right)^2\sqrt{a+c\,x^4}\right)+\left(c^{1/4}\,\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2\,a^{3/4}\,\left(c\,d^2+a\,e^2\right)^2\sqrt{a+c\,x^4}\right)-$$

$$\left(c^{1/4}\,e\,\left(B\,d-A\,e\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2\,a^{1/4}\,d\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(c\,d^2+a\,e^2\right)\sqrt{a+c\,x^4}\right)-$$

$$\left(c^{1/4}\,e\,\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2\,a^{1/4}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(c\,d^2+a\,e^2\right)^2\sqrt{a+c\,x^4}\right)-$$

$$\left(c^{1/4}\,\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2-\frac{\sqrt{c}\,\left(A\,c\,d^2+2\,a\,B\,d\,e-a\,A\,e^2\right)}{\sqrt{a}}\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4\,a^{3/4}\,\left(c\,d^2+a\,e^2\right)^2\sqrt{a+c\,x^4}\right)+$$

$$\left(e\,\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\,\left(B\,d-A\,e\right)\,\left(3\,c\,d^2+a\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a^{1/4}\,c^{1/4}\,d^2\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(c\,d^2+a\,e^2\right)^2\sqrt{a+c\,x^4}\right)+$$

$$\left(a^{3/4}\,e\,\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\,\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4\,c^{1/4}\,d\,\left(c\,d^2-a\,e^2\right)\,\left(c\,d^2+a\,e^2\right)^2\sqrt{a+c\,x^4}\right)$$

Result (type 4, 427 leaves) :

$$\frac{1}{2 a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(c d^3+a d e^2\right)^2 \left(d+e x^2\right) \sqrt{a+c x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} d \left(a e^3 \left(-B d+A e\right) x \left(a+c x^4\right)+c d x \left(d+e x^2\right)\right.\right.$$

$$\left.\left.\left(-a A e^2+B c d^2 x^2+A c d \left(d-2 e x^2\right)+a B e \left(2 d-e x^2\right)\right)\right)-\left(d+e x^2\right) \sqrt{1+\frac{c x^4}{a}}$$

$$\left(-\sqrt{a} \sqrt{c} d \left(-B c d^3+2 A c d^2 e+2 a B d e^2-a A e^3\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]+\right.$$

$$i \left(\sqrt{c} d \left(\sqrt{c} d-i \sqrt{a} e\right) \left(A c d^2+i \sqrt{a} \sqrt{c} d \left(B d-A e\right)+a e \left(2 B d-A e\right)\right)\right.$$

$$\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]+a e \left(-5 B c d^3+7 A c d^2 e+a B d e^2+a A e^3\right)$$

$$\left.\left.\left.\text{EllipticPi}\left[-\frac{i \sqrt{a} e}{\sqrt{c} d}, i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]\right)\right)\right)$$

## Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{\left(d+e x^2\right)^3 \left(a+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 2452 leaves, 22 steps):

$$\left(c x \left(A c d \left(c d^2-3 a e^2\right)+a B e \left(3 c d^2-a e^2\right)+c \left(B c d^3-3 A c d^2 e-3 a B d e^2+a A e^3\right) x^2\right)\right) \Big/$$

$$\left(2 a \left(c d^2+a e^2\right)^3 \sqrt{a+c x^4}\right)+\frac{3 \sqrt{c} e^2 \left(B d-A e\right) \left(3 c d^2+a e^2\right) x \sqrt{a+c x^4}}{8 d^2 \left(c d^2+a e^2\right)^3 \left(\sqrt{a}+\sqrt{c} x^2\right)}+$$

$$\frac{\sqrt{c} e^2 \left(B c d^2-2 A c d e-a B e^2\right) x \sqrt{a+c x^4}}{2 d \left(c d^2+a e^2\right)^3 \left(\sqrt{a}+\sqrt{c} x^2\right)}-$$

$$\frac{c^{3/2} \left(B c d^3-3 A c d^2 e-3 a B d e^2+a A e^3\right) x \sqrt{a+c x^4}}{2 a \left(c d^2+a e^2\right)^3 \left(\sqrt{a}+\sqrt{c} x^2\right)}-\frac{e^3 \left(B d-A e\right) x \sqrt{a+c x^4}}{4 d \left(c d^2+a e^2\right)^2 \left(d+e x^2\right)^2}-$$

$$\frac{3 e^3 \left(B d-A e\right) \left(3 c d^2+a e^2\right) x \sqrt{a+c x^4}}{8 d^2 \left(c d^2+a e^2\right)^3 \left(d+e x^2\right)}-\frac{e^3 \left(B c d^2-2 A c d e-a B e^2\right) x \sqrt{a+c x^4}}{2 d \left(c d^2+a e^2\right)^3 \left(d+e x^2\right)}-$$

$$\frac{e^{3/2}\left(3 c d^2+a e^2\right)\left(B c d^2-2 A c d e-a B e^2\right)\text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\, x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c x^4}}\right]}{4 d^{3/2}\left(c d^2+a e^2\right)^{7/2}}-$$

$$\frac{c\, e^{3/2}\left(B c d^3-3 A c d^2 e-3 a B d e^2+a A e^3\right)\text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\, x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c x^4}}\right]}{2\sqrt{d}\left(c d^2+a e^2\right)^{7/2}}-$$

$$\frac{3 e^{3/2}\left(B d-A e\right)\left(5 c^2 d^4+2 a c d^2 e^2+a^2 e^4\right)\text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\, x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c x^4}}\right]}{16 d^{5/2}\left(c d^2+a e^2\right)^{7/2}}-$$

$$\left(3 a^{1/4} c^{1/4} e^2\left(B d-A e\right)\left(3 c d^2+a e^2\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8 d^2\left(c d^2+a e^2\right)^3\sqrt{a+c x^4}\right)-$$

$$\left(a^{1/4} c^{1/4} e^2\left(B c d^2-2 A c d e-a B e^2\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2 d\left(c d^2+a e^2\right)^3\sqrt{a+c x^4}\right)+$$

$$\left(c^{5/4}\left(B c d^3-3 A c d^2 e-3 a B d e^2+a A e^3\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2 a^{3/4}\left(c d^2+a e^2\right)^3\sqrt{a+c x^4}\right)-$$

$$\left(c^{1/4} e\left(B d-A e\right)\left(4 c d^2-\sqrt{a}\,\sqrt{c}\, d e+3 a e^2\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8 a^{1/4} d^2\left(\sqrt{c}\, d-\sqrt{a}\, e\right)\left(c d^2+a e^2\right)^2\sqrt{a+c x^4}\right)-$$

$$\left(c^{1/4} e\left(B c d^2-2 A c d e-a B e^2\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2\,a^{1/4}\,d\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(c\,d^2+a\,e^2\right)^2\,\sqrt{a+c\,x^4}\right)-$$

$$\left(c^{5/4}\,e\,\left(B\,c\,d^3-3\,A\,c\,d^2\,e-3\,a\,B\,d\,e^2+a\,A\,e^3\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2\,a^{1/4}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(c\,d^2+a\,e^2\right)^3\,\sqrt{a+c\,x^4}\right)+$$

$$\left(c^{3/4}\,\left(A\,c^2\,d^3-a^2\,B\,e^3-\sqrt{a}\,c^{3/2}\,d^2\,\left(B\,d-3\,A\,e\right)+3\,a\,c\,d\,e\,\left(B\,d-A\,e\right)+a^{3/2}\,\sqrt{c}\,e^2\,\left(3\,B\,d-A\,e\right)\right)\right.$$

$$\left.\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(4\,a^{5/4}\,\left(c\,d^2+a\,e^2\right)^3\,\sqrt{a+c\,x^4}\right)+\left(e\,\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\,\left(3\,c\,d^2+a\,e^2\right)\,\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)\right.$$

$$\left.\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a^{1/4}\,c^{1/4}\,d^2\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(c\,d^2+a\,e^2\right)^3\,\sqrt{a+c\,x^4}\right)+$$

$$\left(a^{3/4}\,c^{3/4}\,e\,\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\,\left(B\,c\,d^3-3\,A\,c\,d^2\,e-3\,a\,B\,d\,e^2+a\,A\,e^3\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(4\,d\,\left(c\,d^2-a\,e^2\right)\,\left(c\,d^2+a\,e^2\right)^3\,\sqrt{a+c\,x^4}\right)+$$

$$\left(3\,e\,\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\,\left(B\,d-A\,e\right)\,\left(5\,c^2\,d^4+2\,a\,c\,d^2\,e^2+a^2\,e^4\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\, a^{1/4}\, c^{1/4}\, d^3 \left(\sqrt{c}\, d - \sqrt{a}\, e\right) \left(c\, d^2 + a\, e^2\right)^3 \sqrt{a + c\, x^4}\right)$$

Result (type 4, 630 leaves):

$$\frac{1}{8\, a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(c\, d^3 + a\, d\, e^2\right)^3 \left(d + e\, x^2\right)^2 \sqrt{a + c\, x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, d\, x \left(-2\, a\, d\, e^3 \left(B\, d - A\, e\right) \left(c\, d^2 + a\, e^2\right) \left(a + c\, x^4\right) + a\, e^3 \left(-13\, B\, c\, d^3 + 17\, A\, c\, d^2\, e + a\, B\, d\, e^2 + 3\, a\, A\, e^3\right) \left(d + e\, x^2\right) \left(a + c\, x^4\right) + 4\, c\, d^2 \left(d + e\, x^2\right)^2 \left(B \left(-a^2\, e^3 + c^2\, d^3\, x^2 + 3\, a\, c\, d\, e \left(d - e\, x^2\right)\right) + A\, c \left(c\, d^2 \left(d - 3\, e\, x^2\right) + a\, e^2 \left(-3\, d + e\, x^2\right)\right)\right)\right) -\right.$$

$$\left(d + e\, x^2\right)^2 \sqrt{1 + \frac{c\, x^4}{a}} \left(\sqrt{a}\, \sqrt{c}\, d \left(3\, A\, e \left(-4\, c^2\, d^4 + 7\, a\, c\, d^2\, e^2 + a^2\, e^4\right) + B \left(4\, c^2\, d^5 - 25\, a\, c\, d^3\, e^2 + a^2\, d\, e^4\right)\right) \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right], -1\right] + \right.$$

$$i \left(\sqrt{c}\, d \left(\sqrt{c}\, d - i \sqrt{a}\, e\right) \left(4\, A\, c^2\, d^4 + 4\, i \sqrt{a}\, c^{3/2}\, d^3 \left(B\, d - 2\, A\, e\right) + 19\, a\, c\, d^2\, e \left(B\, d - A\, e\right) - 2\, i\, a^{3/2} \sqrt{c}\, d\, e^2 \left(3\, B\, d - A\, e\right) - a^2\, e^3 \left(B\, d + 3\, A\, e\right)\right) \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right], -1\right] + a\, e \left(3\, A\, e \left(21\, c^2\, d^4 + 2\, a\, c\, d^2\, e^2 + a^2\, e^4\right) + B \left(-35\, c^2\, d^5 + 26\, a\, c\, d^3\, e^2 + a^2\, d\, e^4\right)\right)\right.$$

$$\left.\left.\left.\text{EllipticPi}\left[-\frac{i \sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right], -1\right]\right)\right)\right)$$

## Problem 15: Unable to integrate problem.

$$\int \frac{\left(A + B\, x^2\right) \left(d + e\, x^2\right)^q}{a + c\, x^4}\, dx$$

Optimal (type 6, 169 leaves, 6 steps):

$$\frac{1}{2\, a}\left(A - \frac{\sqrt{-a}\, B}{\sqrt{c}}\right) x \left(d + e\, x^2\right)^q \left(1 + \frac{e\, x^2}{d}\right)^{-q} \text{AppellF1}\left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{\sqrt{c}\, x^2}{\sqrt{-a}}, -\frac{e\, x^2}{d}\right] + \frac{1}{2\, a}\left(A + \frac{\sqrt{-a}\, B}{\sqrt{c}}\right) x \left(d + e\, x^2\right)^q \left(1 + \frac{e\, x^2}{d}\right)^{-q} \text{AppellF1}\left[\frac{1}{2}, 1, -q, \frac{3}{2}, \frac{\sqrt{c}\, x^2}{\sqrt{-a}}, -\frac{e\, x^2}{d}\right]$$

Result (type 8, 28 leaves):

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)^q}{a+c x^4} \, dx$$

## Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+x^2}{\left(1+x^2\right) \sqrt{2+3 x^2+x^4}} \, dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\frac{\sqrt{2} \left(2+x^2\right) \text{EllipticE}\left[\text{ArcTan}[x], \frac{1}{2}\right]}{\sqrt{\frac{2+x^2}{1+x^2}} \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 94 leaves):

$$\frac{1}{\sqrt{2+3 x^2+x^4}} \left(2 x+x^3+i \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right], 2\right] - i \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right], 2\right]\right)$$

## Problem 17: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)^3}{\sqrt{a+b x^2+c x^4}} \, dx$$

Optimal (type 4, 755 leaves, 6 steps):

$$\frac{1}{105\,c^3}e\,\left(7\,A\,c\,e\,\left(15\,c\,d-4\,b\,e\right)+B\,\left(105\,c^2\,d^2+24\,b^2\,e^2-c\,e\,\left(84\,b\,d+25\,a\,e\right)\right)\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}\,+$$

$$\frac{e^2\,\left(21\,B\,c\,d-6\,b\,B\,e+7\,A\,c\,e\right)\,x^3\,\sqrt{a+b\,x^2+c\,x^4}}{35\,c^2}+\frac{B\,e^3\,x^5\,\sqrt{a+b\,x^2+c\,x^4}}{7\,c}+$$

$$\left(\left(7\,A\,c\,e\,\left(45\,c^2\,d^2+8\,b^2\,e^2-3\,c\,e\,\left(10\,b\,d+3\,a\,e\right)\right)+B\right.\right.$$

$$\left.\left.\left(105\,c^3\,d^3-48\,b^3\,e^3-21\,c^2\,d\,e\,\left(10\,b\,d+9\,a\,e\right)+8\,b\,c\,e^2\,\left(21\,b\,d+13\,a\,e\right)\right)\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}\right)\Big/$$

$$\left(105\,c^{7/2}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\right)-\left(a^{1/4}\,\left(7\,A\,c\,e\,\left(45\,c^2\,d^2+8\,b^2\,e^2-3\,c\,e\,\left(10\,b\,d+3\,a\,e\right)\right)+\right.\right.$$

$$\left.B\,\left(105\,c^3\,d^3-48\,b^3\,e^3-21\,c^2\,d\,e\,\left(10\,b\,d+9\,a\,e\right)+8\,b\,c\,e^2\,\left(21\,b\,d+13\,a\,e\right)\right)\right)$$

$$\left.\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(105\,c^{15/4}\,\sqrt{a+b\,x^2+c\,x^4}\right)+\left(a^{1/4}\,\left(7\,A\,c\,e\,\left(45\,c^2\,d^2+8\,b^2\,e^2-3\,c\,e\,\left(10\,b\,d+3\,a\,e\right)\right)+\right.\right.$$

$$B\,\left(105\,c^3\,d^3-48\,b^3\,e^3-21\,c^2\,d\,e\,\left(10\,b\,d+9\,a\,e\right)+8\,b\,c\,e^2\,\left(21\,b\,d+13\,a\,e\right)\right)+\frac{1}{\sqrt{a}}$$

$$\left.\sqrt{c}\,\left(7\,A\,c\,\left(15\,c^2\,d^3-15\,a\,c\,d\,e^2+4\,a\,b\,e^3\right)-a\,B\,e\,\left(105\,c^2\,d^2+24\,b^2\,e^2-c\,e\,\left(84\,b\,d+25\,a\,e\right)\right)\right)\right)$$

$$\left.\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(210\,c^{15/4}\,\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 4473 leaves):

$$\sqrt{a+b\,x^2+c\,x^4}$$

$$\left(-\frac{1}{105\,c^3}e\,\left(-105\,B\,c^2\,d^2+84\,b\,B\,c\,d\,e-105\,A\,c^2\,d\,e-24\,b^2\,B\,e^2+28\,A\,b\,c\,e^2+25\,a\,B\,c\,e^2\right)\,x+\right.$$

$$\left.\frac{e^2\,\left(21\,B\,c\,d-6\,b\,B\,e+7\,A\,c\,e\right)\,x^3}{35\,c^2}+\frac{B\,e^3\,x^5}{7\,c}\right)+$$

$$\frac{1}{105\,c^3}\left(\left(\left(105\,\mathrm{i}\,B\,c^2\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.\right.\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Bigg(105\,\text{i}\,b\,B\,c\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\Bigg(315\,\text{i}\,A\,c^2\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\Bigg(42\,\text{i}\,\sqrt{2}\,b^2\,B\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(105\,\mathrm{i}\,A\,b\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(189\,\mathrm{i}\,a\,B\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(14\,\mathrm{i}\sqrt{2}\,A\,b^2\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(26\,\mathrm{i}\sqrt{2}\,a\,b\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(12\,\mathbb{i}\,\sqrt{2}\,b^3\,B\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Bigg/$$

$$\left(c\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(63\,\mathbb{i}\,a\,A\,c\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Bigg/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(105\,\mathbb{i}\,A\,c^3\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(105\,\mathbb{i}\,a\,B\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(42\,\text{i}\,\sqrt{2}\,a\,b\,B\,c\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(105\,\text{i}\,a\,A\,c^2\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(12\,\text{i}\,\sqrt{2}\,a\,b^2\,B\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(14\,\text{i}\,\sqrt{2}\,a\,A\,b\,c\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(25\, i\, a^2 B c e^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$
$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$
$$\left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right)\Bigg)$$

## Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)^2}{\sqrt{a+b x^2+c x^4}}\, dx$$

Optimal (type 4, 528 leaves, 5 steps):

$$\frac{e\,(10 B c d-4 b B e+5 A c e)\, x \sqrt{a+b x^2+c x^4}}{15 c^2}+\frac{B e^2 x^3 \sqrt{a+b x^2+c x^4}}{5 c}+$$
$$\left(\left(10 A c e\,(3 c d-b e)+B\left(15 c^2 d^2+8 b^2 e^2-c e\,(20 b d+9 a e)\right)\right) x \sqrt{a+b x^2+c x^4}\right)\Big/$$
$$\left(15 c^{5/2}\left(\sqrt{a}+\sqrt{c}\, x^2\right)\right)-\left(a^{1/4}\left(10 A c e\,(3 c d-b e)+B\left(15 c^2 d^2+8 b^2 e^2-c e\,(20 b d+9 a e)\right)\right)\right.$$
$$\left.\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\; \text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/$$
$$\left(15 c^{11/4} \sqrt{a+b x^2+c x^4}\right)+\left(a^{1/4}\left(10 A c e\,(3 c d-b e)+B\left(15 c^2 d^2+8 b^2 e^2-c e\,(20 b d+9 a e)\right)-\right.\right.$$
$$\left.\frac{\sqrt{c}\left(2 a B e\,(5 c d-2 b e)-5 A c\left(3 c d^2-a e^2\right)\right)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}$$
$$\left.\text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(30 c^{11/4} \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 2613 leaves):

$$\left(\frac{e\,(10 B c d-4 b B e+5 A c e)\, x}{15 c^2}+\frac{B e^2 x^3}{5 c}\right)\sqrt{a+b x^2+c x^4}+$$

$$\frac{1}{15\,c^2}\left(\left(15\,\mathrm{i}\,B\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(5\,\mathrm{i}\sqrt{2}\,b\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(15\,\mathrm{i}\,A\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(5\,\mathrm{i}\,A\,b\left(-b+\sqrt{b^2-4\,a\,c}\right)e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(9\,\text{i}\,a\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(2\,\text{i}\sqrt{2}\,b^2\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(c\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(15\,\text{i}\,A\,c^2\,d^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(5\,\text{i}\sqrt{2}\,a\,B\,c\,d\,e\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(2\,\mathrm{i}\,\sqrt{2}\,a\,b\,B\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(5\,\mathrm{i}\,a\,A\,c\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)\Bigg)$$

## Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 368 leaves, 4 steps):

$$\frac{B\,e\,x\,\sqrt{a+b\,x^2+c\,x^4}}{3\,c}+\frac{\left(3\,B\,c\,d-2\,b\,B\,e+3\,A\,c\,e\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{3\,c^{3/2}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-$$
$$\left(a^{1/4}\,\left(3\,B\,c\,d-2\,b\,B\,e+3\,A\,c\,e\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$
$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(3\,c^{7/4}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$
$$\left(a^{1/4}\,\left(3\,B\,c\,d-2\,b\,B\,e+3\,A\,c\,e+\frac{\sqrt{c}\,\left(3\,A\,c\,d-a\,B\,e\right)}{\sqrt{a}}\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(6\,c^{7/4}\,\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 521 leaves):

$$\frac{1}{12\,c^2\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}$$
$$\left(4\,B\,c\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,e\,x\,\left(a+b\,x^2+c\,x^4\right)-\mathrm{i}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\left(-3\,B\,c\,d+2\,b\,B\,e-3\,A\,c\,e\right)\right.$$
$$\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$
$$\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$
$$\mathrm{i}\,\left(-2\,b^2\,B\,e-c\,\left(6\,A\,c\,d+3\,B\,\sqrt{b^2-4\,a\,c}\,d-2\,a\,B\,e+3\,A\,\sqrt{b^2-4\,a\,c}\,e\right)+\right.$$
$$\left.b\,\left(3\,B\,c\,d+3\,A\,c\,e+2\,B\,\sqrt{b^2-4\,a\,c}\,e\right)\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$
$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

# Problem 20: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{\sqrt{a + b x^2 + c x^4}} \, dx$$

Optimal (type 4, 283 leaves, 3 steps):

$$\frac{B x \sqrt{a + b x^2 + c x^4}}{\sqrt{c} \left(\sqrt{a} + \sqrt{c} x^2\right)} - \left(a^{1/4} B \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(c^{3/4} \sqrt{a + b x^2 + c x^4}\right) + \left(a^{1/4} \left(B + \frac{A \sqrt{c}}{\sqrt{a}}\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(2 c^{3/4} \sqrt{a + b x^2 + c x^4}\right)$$

Result (type 4, 302 leaves):

$$\left(i \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \left(B \left(-b + \sqrt{b^2 - 4 a c}\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] + \left(b B - 2 A c - B \sqrt{b^2 - 4 a c}\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right]\right)\right) \Big/ \left(2 \sqrt{2} c \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4}\right)$$

# Problem 21: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{\left(d + e x^2\right) \sqrt{a + b x^2 + c x^4}} \, dx$$

Optimal (type 4, 436 leaves, 3 steps):

$$-\frac{(B d-A e)\,\text{ArcTan}\left[\frac{\sqrt{c d^2-b d e+a e^2}\, x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+b x^2+c x^4}}\right]}{2\sqrt{d}\,\sqrt{e}\,\sqrt{c d^2-b d e+a e^2}}-$$

$$\left(\left(\sqrt{a}\,B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\right.\right.$$

$$\left.\left.\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(2\,a^{1/4} c^{1/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+b x^2+c x^4}\right)+$$

$$\left(a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2 (B d-A e)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,d e},\right.\right.$$

$$\left.\left.2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(4\,c^{1/4} d e\left(c d^2-a e^2\right)\sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 298 leaves):

$$-\left(\left(i\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\right.\right.$$

$$\left(B d\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\,x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+\right.$$

$$(-B d+A e)\,\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4 a c}\right) e}{2 c d},i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\,x\right],\right.$$

$$\left.\left.\left.\left.\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)\right)\Big/\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\,d e\sqrt{a+b x^2+c x^4}\right)\right)$$

## Problem 22: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{A+B x^2}{\left(d+e x^2\right)^2\sqrt{a+b x^2+c x^4}}\,dx$$

Optimal (type 4, 782 leaves, 6 steps):

$$\frac{\sqrt{c}\,(B d-A e)\,x\,\sqrt{a+b x^2+c x^4}}{2 d\,(c d^2-b d e+a e^2)\,(\sqrt{a}+\sqrt{c}\,x^2)}-\frac{e\,(B d-A e)\,x\,\sqrt{a+b x^2+c x^4}}{2 d\,(c d^2-b d e+a e^2)\,(d+e x^2)}-$$

$$\left(\left(B\,(c d^3-a d e^2)-A e\,(3 c d^2-e\,(2 b d-a e))\right)\text{ArcTan}\left[\frac{\sqrt{c d^2-b d e+a e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+b x^2+c x^4}}\right]\right)\Big/$$

$$\left(4 d^{3/2}\sqrt{e}\,(c d^2-b d e+a e^2)^{3/2}\right)-\left(a^{1/4} c^{1/4}\,(B d-A e)\,(\sqrt{a}+\sqrt{c}\,x^2)\,\sqrt{\frac{a+b x^2+c x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(2 d\,(c d^2-b d e+a e^2)\,\sqrt{a+b x^2+c x^4}\right)+$$

$$\left(A c^{1/4}\,(\sqrt{a}+\sqrt{c}\,x^2)\,\sqrt{\frac{a+b x^2+c x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(2 a^{1/4} d\,(\sqrt{c}\,d-\sqrt{a}\,e)\,\sqrt{a+b x^2+c x^4}\right)+$$

$$\left((\sqrt{c}\,d+\sqrt{a}\,e)\,\left(B\,(c d^3-a d e^2)-A e\,(3 c d^2-e\,(2 b d-a e))\right)\,(\sqrt{a}+\sqrt{c}\,x^2)\right.$$

$$\left.\sqrt{\frac{a+b x^2+c x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\,\text{EllipticPi}\left[-\frac{(\sqrt{c}\,d-\sqrt{a}\,e)^2}{4\sqrt{a}\,\sqrt{c}\,d e},2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(8 a^{1/4} c^{1/4} d^2 e\,(\sqrt{c}\,d-\sqrt{a}\,e)\,(c d^2-b d e+a e^2)\,\sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 2187 leaves):

$$-\frac{e\,(B d-A e)\,x\,\sqrt{a+b x^2+c x^4}}{2 d\,(c d^2-b d e+a e^2)\,(d+e x^2)}+$$

$$\frac{1}{2 d\,(c d^2-b d e+a e^2)}\left(\left(\left(i B\,\left(-b+\sqrt{b^2-4 a c}\right)d\,\sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\,\sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,x\right],\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,x\right],\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,\sqrt{a+b x^2+c x^4}\right)-$$

$$\left(\mathbb{i}\,A\left(-b+\sqrt{b^2-4ac}\right)e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+$$

$$\left(\mathbb{i}\,A\,c\,d\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\left(\mathbb{i}\,B\,c\,d^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,e\sqrt{a+bx^2+cx^4}\right)-$$

$$\left(3\,\mathbb{i}\,A\,c\,d\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+$$

$$\left(\mathbb{i}\,B\,c\,d^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticPi}\left[\right.\right.$$

$$-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\ \mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\ \frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Bigg]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,e\sqrt{a+bx^2+cx^4}\right)+$$

$$\left(\mathrm{i}\sqrt{2}\,A\,b\,e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\mathrm{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\ \mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\right.$$

$$\left.\left.\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Bigg/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\left(\mathrm{i}\,a\,B\,e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\mathrm{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\right.\right.$$

$$\left.\left.\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\ \frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\left(\mathrm{i}\,a\,A\,e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\mathrm{EllipticPi}\Bigg[\right.$$

$$\left.-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\ \mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\ \frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,d\sqrt{a+bx^2+cx^4}\right)\Bigg)$$

## Problem 23: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{A+Bx^2}{\left(d+ex^2\right)^3\sqrt{a+bx^2+cx^4}}\,dx$$

Optimal (type 4, 1125 leaves, 7 steps):

$$-\left(\left(\sqrt{c}\,\left(3\,A\,e\,\left(3\,c\,d^2-e\,\left(2\,b\,d-a\,e\right)\right)-B\,d\,\left(5\,c\,d^2-e\,\left(2\,b\,d+a\,e\right)\right)\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}\right)\Big/\left(8\,d^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\right)\right)-\frac{e\,\left(B\,d-A\,e\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{4\,d\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\left(d+e\,x^2\right)^2}+$$

$$\left(e\,\left(3\,A\,e\,\left(3\,c\,d^2-e\,\left(2\,b\,d-a\,e\right)\right)-B\,d\,\left(5\,c\,d^2-e\,\left(2\,b\,d+a\,e\right)\right)\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}\right)\Big/\left(8\,d^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\left(d+e\,x^2\right)\right)-\left(\left(B\,d\,\left(3\,c^2\,d^4-10\,a\,c\,d^2\,e^2+a\,e^3\,\left(4\,b\,d-a\,e\right)\right)-A\,e\,\left(15\,c^2\,d^4-2\,c\,d^2\,e\,\left(10\,b\,d-3\,a\,e\right)+e^2\,\left(8\,b^2\,d^2-8\,a\,b\,d\,e+3\,a^2\,e^2\right)\right)\right)\,\text{ArcTan}\left[\frac{\sqrt{c\,d^2-b\,d\,e+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+b\,x^2+c\,x^4}}\right]\right)\Big/\left(16\,d^{5/2}\,\sqrt{e}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^{5/2}\right)+$$

$$\left(a^{1/4}\,c^{1/4}\,\left(3\,A\,e\,\left(3\,c\,d^2-e\,\left(2\,b\,d-a\,e\right)\right)-B\,d\,\left(5\,c\,d^2-e\,\left(2\,b\,d+a\,e\right)\right)\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(8\,d^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(c^{1/4}\,\left(\sqrt{a}\,\sqrt{c}\,d\,\left(B\,d-A\,e\right)+a\,e\,\left(B\,d+3\,A\,e\right)+4\,A\,d\,\left(c\,d-b\,e\right)\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(8\,a^{1/4}\,d^2\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\,\left(B\,d\,\left(3\,c^2\,d^4-10\,a\,c\,d^2\,e^2+a\,e^3\,\left(4\,b\,d-a\,e\right)\right)-A\,e\,\left(15\,c^2\,d^4-2\,c\,d^2\,e\,\left(10\,b\,d-3\,a\,e\right)+e^2\,\left(8\,b^2\,d^2-8\,a\,b\,d\,e+3\,a^2\,e^2\right)\right)\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(32\,a^{1/4}\,c^{1/4}\,d^3\,e\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 5205 leaves):

$$\sqrt{a+b\,x^2+c\,x^4}\,\left(-\frac{e\,\left(B\,d-A\,e\right)\,x}{4\,d\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\left(d+e\,x^2\right)^2}-\right.$$

$$\left.\frac{e\left(5\,B\,c\,d^3-2\,b\,B\,d^2\,e-9\,A\,c\,d^2\,e+6\,A\,b\,d\,e^2-a\,B\,d\,e^2-3\,a\,A\,e^3\right)x}{8\,d^2\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\left(d+e\,x^2\right)}\right)+\frac{1}{8\,d^2\left(c\,d^2-b\,d\,e+a\,e^2\right)^2}$$

$$\left(\left(\left(5\,\mathrm{i}\,B\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.\right.\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\mathrm{i}\,b\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2\,e\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(9\,\mathrm{i}\,A\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2\,e\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(3\,\mathrm{i}\,A\,b\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\text{i}\,a\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Bigg/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(3\,\text{i}\,a\,A\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Bigg/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(7\,\text{i}\,A\,c^2\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(3\,\mathrm{i}\,B\,c^2\,d^4\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;e\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(2\,\mathrm{i}\,\sqrt{2}\,A\,b\,c\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(3\,\mathrm{i}\,a\,B\,c\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(\mathrm{i}\,a\,A\,c\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(15\,\mathrm{i}\,A\,c^2\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\;\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(3\,\mathbb{i}\,B\,c^2\,d^4\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)\,e}{2\,c\,d},\,\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,e\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(10\,\mathbb{i}\,\sqrt{2}\,A\,b\,c\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)\,e}{2\,c\,d},\,\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(5\,\mathbb{i}\,\sqrt{2}\,a\,B\,c\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)\,e}{2\,c\,d},\,\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(4\,\mathbb{i}\,\sqrt{2}\,A\,b^2\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)\,e}{2\,c\,d},\,\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,a\,b\,B\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\right.$$

$$\left.\left.\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(3\,\mathrm{i}\,\sqrt{2}\,a\,A\,c\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\right.$$

$$\left.\left.\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(4\,\mathrm{i}\,\sqrt{2}\,a\,A\,b\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\right.$$

$$\left.\left.\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\mathrm{i}\,a^2\,B\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\right.\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(3\,\mathrm{i}\,a^2\,A\,e^4\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left.\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,d\,\sqrt{a+b\,x^2+c\,x^4}\right)\right)$$

## Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)^3}{\left(a+b x^2+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 859 leaves, 5 steps):

$$\left(x \left(A c \left(b^2 c d^3-2 a c d \left(c d^2-3 a e^2\right)-a b e \left(3 c d^2+a e^2\right)\right)+a B \left(a b^2 e^3+2 a c e \left(3 c d^2-a e^2\right)-b c d \left(c d^2+3 a e^2\right)\right)-\left(a B \left(2 c d-b e\right) \left(c^2 d^2+b^2 e^2-c e \left(b d+3 a e\right)\right)+A c \left(a b^2 e^3+2 a c e \left(3 c d^2-a e^2\right)-b c d \left(c d^2+3 a e^2\right)\right)\right) x^2\right)\right) \Big/ \left(a c^2 \left(b^2-4 a c\right) \sqrt{a+b x^2+c x^4}\right)+\frac{B e^3 x \sqrt{a+b x^2+c x^4}}{3 c^2}+\left(\left(a B \left(6 c^3 d^3-8 b^3 e^3-9 c^2 d e \left(b d+6 a e\right)+b c e^2 \left(18 b d+29 a e\right)\right)+3 A c \left(2 a b^2 e^3+6 a c e \left(c d^2-a e^2\right)-b c d \left(c d^2+3 a e^2\right)\right)\right) x \sqrt{a+b x^2+c x^4}\right) \Big/ \left(3 a c^{5/2} \left(b^2-4 a c\right) \left(\sqrt{a}+\sqrt{c} x^2\right)\right)-\left(\left(a B \left(6 c^3 d^3-8 b^3 e^3-9 c^2 d e \left(b d+6 a e\right)+b c e^2 \left(18 b d+29 a e\right)\right)+3 A c \left(2 a b^2 e^3+6 a c e \left(c d^2-a e^2\right)-b c d \left(c d^2+3 a e^2\right)\right)\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(3 a^{3/4} c^{11/4} \left(b^2-4 a c\right) \sqrt{a+b x^2+c x^4}\right)-\left(\left(3 A c^3 d^3-5 a^2 B c e^3-3 \sqrt{a} c^{5/2} d^2 \left(B d+3 A e\right)+a e \left(3 c d-2 b e\right) \left(3 B c d-4 b B e+3 A c e\right)+3 a^{3/2} \sqrt{c} e^2 \left(9 B c d-4 b B e+3 A c e\right)\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(6 a^{3/4} \left(b-2 \sqrt{a} \sqrt{c}\right) c^{11/4} \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 5432 leaves):

$$\sqrt{a+b x^2+c x^4} \left(\frac{B e^3 x}{3 c^2}+\frac{1}{a c^2 \left(-b^2+4 a c\right) \left(a+b x^2+c x^4\right)}\right.$$
$$\left(-A b^2 c^2 d^3 x+a b B c^2 d^3 x+2 a A c^3 d^3 x+3 a A b c^2 d^2 e x-6 a^2 B c^2 d^2 e x+3 a^2 b B c d e^2 x-\right.$$

$$6 a^2 A c^2 d e^2 x - a^2 b^2 B e^3 x + a^2 A b c e^3 x + 2 a^3 B c e^3 x - A b c^3 d^3 x^3 + 2 a B c^3 d^3 x^3 - 3 a b B c^2 d^2 e x^3 + 6 a A c^3 d^2 e x^3 + 3 a b^2 B c d e^2 x^3 - 3 a A b c^2 d e^2 x^3 - 6 a^2 B c^2 d e^2 x^3 - a b^3 B e^3 x^3 + a A b^2 c e^3 x^3 + 3 a^2 b B c e^3 x^3 - 2 a^2 A c^2 e^3 x^3\Big)\Bigg) - \frac{1}{3 a c^2 \left(-b^2 + 4 a c\right)}$$

$$\Bigg(-\Bigg(\Bigg(3 \mathrm{i} A b c^2 \left(-b + \sqrt{b^2 - 4 a c}\right) d^3 \sqrt{1 - \frac{2 c x^2}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}}$$

$$\Bigg(\text{EllipticE}\Big[\mathrm{i} \text{ArcSinh}\Big[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\Big], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\Big] -$$

$$\text{EllipticF}\Big[\mathrm{i} \text{ArcSinh}\Big[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\Big], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\Big]\Bigg)\Bigg) \Big/$$

$$\left(2 \sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4}\right)\Bigg) +$$

$$\Bigg(3 \mathrm{i} a B c^2 \left(-b + \sqrt{b^2 - 4 a c}\right) d^3 \sqrt{1 - \frac{2 c x^2}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}}$$

$$\Bigg(\text{EllipticE}\Big[\mathrm{i} \text{ArcSinh}\Big[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\Big], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\Big] -$$

$$\text{EllipticF}\Big[\mathrm{i} \text{ArcSinh}\Big[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\Big], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\Big]\Bigg)\Bigg) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4}\right) -$$

$$\Bigg(9 \mathrm{i} a b B c \left(-b + \sqrt{b^2 - 4 a c}\right) d^2 e \sqrt{1 - \frac{2 c x^2}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}}$$

$$\Bigg(\text{EllipticE}\Big[\mathrm{i} \text{ArcSinh}\Big[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\Big], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\Big] -$$

$$\text{EllipticF}\Big[\mathrm{i} \text{ArcSinh}\Big[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\Big], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\Big]\Bigg)\Bigg) \Big/$$

$$\left(2 \sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4}\right) +$$

$$\left(9\,\mathrm{i}\,a\,A\,c^2\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(9\,\mathrm{i}\,a\,b^2\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(9\,\mathrm{i}\,a\,A\,b\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(27\,\mathrm{i}\,a^2\,B\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\Bigg(3\,\mathbb{i}\,a\,A\,b^2\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\Bigg(29\,\mathbb{i}\,a^2\,b\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Bigg(2\,\mathbb{i}\,\sqrt{2}\,a\,b^3\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(c\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(9\,\mathrm{i}\,a^2\,A\,c\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(3\,\mathrm{i}\,a\,b\,B\,c^2\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(3\,\mathrm{i}\,\sqrt{2}\,a\,A\,c^3\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(9\,\mathrm{i}\,a\,A\,b\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(9\,\mathrm{i}\,\sqrt{2}\,a^2\,B\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(9\,\mathbb{i}\,a^2\,b\,B\,c\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(9\,\mathbb{i}\,\sqrt{2}\,a^2\,A\,c^2\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,a^2\,b^2\,B\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(3\,\mathbb{i}\,a^2\,A\,b\,c\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(5\, i \sqrt{2}\, a^3 B c e^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$
$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$
$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right)\Bigg)$$

## Problem 25: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)^2}{\left(a+b x^2+c x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 628 leaves, 4 steps):

$$-\left(\left(x \left(a B \left(b c d^2-4 a c d e+a b e^2\right)-A c \left(b^2 d^2-2 a b d e-2 a \left(c d^2-a e^2\right)\right)\right)-\right.\right.$$
$$\left.\left.\left(A c \left(b c d^2-4 a c d e+a b e^2\right)-a B \left(2 c^2 d^2+b^2 e^2-2 c e \left(b d+a e\right)\right)\right) x^2\right)\right) \Big/$$
$$\left.\left(a c \left(b^2-4 a c\right) \sqrt{a+b x^2+c x^4}\right)\right) -$$
$$\left(\left(A c \left(b c d^2-4 a c d e+a b e^2\right)-2 a B \left(c^2 d^2+b^2 e^2-c e \left(b d+3 a e\right)\right)\right) x \sqrt{a+b x^2+c x^4}\right) \Big/$$
$$\left(a c^{3/2} \left(b^2-4 a c\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right)\right)+$$
$$\left(\left(A c \left(b c d^2-4 a c d e+a b e^2\right)-2 a B \left(c^2 d^2+b^2 e^2-c e \left(b d+3 a e\right)\right)\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right)\right.$$
$$\left.\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/$$
$$\left(a^{3/4} c^{7/4} \left(b^2-4 a c\right) \sqrt{a+b x^2+c x^4}\right) -$$
$$\left(\left(A c^2 d^2+3 a^{3/2} B \sqrt{c}\, e^2-\sqrt{a}\, c^{3/2} d \left(B d+2 A e\right)+a e \left(2 B c d-2 b B e+A c e\right)\right)\right.$$
$$\left.\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/$$
$$\left(2 a^{3/4} \left(b-2 \sqrt{a} \sqrt{c}\right) c^{7/4} \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 3464 leaves):

$$\left(-A b^2 c d^2 x+a b B c d^2 x+2 a A c^2 d^2 x+2 a A b c d e x-4 a^2 B c d e x+a^2 b B e^2 x-2 a^2 A c e^2 x-A b c^2 d^2 x^3+2 a B c^2 d^2 x^3-2 a b B c d e x^3+4 a A c^2 d e x^3+a b^2 B e^2 x^3-a A b c e^2 x^3-2 a^2 B c e^2 x^3\right)\Big/\left(a c\left(-b^2+4 a c\right)\sqrt{a+b x^2+c x^4}\right)-$$

$$\frac{1}{a c\left(-b^2+4 a c\right)}\left(-\left(\left(\left(\mathrm{i} A b c\left(-b+\sqrt{b^2-4 a c}\right) d^2\sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.\right.\right.\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,x\right],\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,x\right],\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\Big/$$

$$\left.\left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\sqrt{a+b x^2+c x^4}\right)\right)+$$

$$\left(\mathrm{i} a B c\left(-b+\sqrt{b^2-4 a c}\right) d^2\sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,x\right],\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,x\right],\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\sqrt{a+b x^2+c x^4}\right)-$$

$$\left(\mathrm{i} a b B\left(-b+\sqrt{b^2-4 a c}\right) d e\sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,x\right],\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,x\right],\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\sqrt{a+b x^2+c x^4}\right)+$$

$$\left(i \sqrt{2}\, a A c \left(-b+\sqrt{b^2-4 a c}\right) d e \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right) \Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\left(i\, a A b \left(-b+\sqrt{b^2-4 a c}\right) e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\left(3\, i\, a^2 B \left(-b+\sqrt{b^2-4 a c}\right) e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) +$$

$$\left(i\, a b^2 B \left(-b+\sqrt{b^2-4 a c}\right) e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right] -\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,c\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Bigg(\mathbb{i}\,a\,b\,B\,c\,d^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\Bigg(\mathbb{i}\,\sqrt{2}\,a\,A\,c^2\,d^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Bigg(\mathbb{i}\,\sqrt{2}\,a\,A\,b\,c\,d\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\Bigg(2\,\mathbb{i}\,\sqrt{2}\,a^2\,B\,c\,d\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(i\, a^2\, b\, B\, e^2 \sqrt{1-\frac{2\, c\, x^2}{-b-\sqrt{b^2-4\, a\, c}}}\, \sqrt{1-\frac{2\, c\, x^2}{-b+\sqrt{b^2-4\, a\, c}}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2}\, \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\, x\right], \frac{-b-\sqrt{b^2-4\, a\, c}}{-b+\sqrt{b^2-4\, a\, c}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\, \sqrt{a+b\, x^2+c\, x^4}\right) +$$

$$\left(i\, \sqrt{2}\, a^2\, A\, c\, e^2 \sqrt{1-\frac{2\, c\, x^2}{-b-\sqrt{b^2-4\, a\, c}}}\, \sqrt{1-\frac{2\, c\, x^2}{-b+\sqrt{b^2-4\, a\, c}}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2}\, \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\, x\right], \frac{-b-\sqrt{b^2-4\, a\, c}}{-b+\sqrt{b^2-4\, a\, c}}\right]\right) \Big/$$

$$\left.\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\, \sqrt{a+b\, x^2+c\, x^4}\right)\right)$$

## Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B\, x^2\right)\, \left(d+e\, x^2\right)}{\left(a+b\, x^2+c\, x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 481 leaves, 4 steps):

$$-\left(\left(x\left(a B\left(b d-2 a e\right)-A\left(b^2 d-2 a c d-a b e\right)-\left(A c\left(b d-2 a e\right)-a B\left(2 c d-b e\right)\right) x^2\right)\right)\Big/\left(a\left(b^2-4 a c\right)\sqrt{a+b x^2+c x^4}\right)\right)-\frac{\left(A c\left(b d-2 a e\right)-a B\left(2 c d-b e\right)\right) x\sqrt{a+b x^2+c x^4}}{a\sqrt{c}\left(b^2-4 a c\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)}+$$

$$\left(\left(A c\left(b d-2 a e\right)-a B\left(2 c d-b e\right)\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(a^{3/4} c^{3/4}\left(b^2-4 a c\right)\sqrt{a+b x^2+c x^4}\right)+$$

$$\left(\left(\sqrt{a}\, B-A\sqrt{c}\right)\left(\sqrt{c}\, d-\sqrt{a}\, e\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(2 a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right) c^{3/4}\sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 597 leaves):

$$\frac{1}{4 a c\left(-b^2+4 a c\right)\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\sqrt{a+b x^2+c x^4}}\left(4 c\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right.$$

$$\left(a B\left(-2 a e+2 c d x^2+b\left(d-e x^2\right)\right)+A\left(-b^2 d+b\left(a e-c d x^2\right)+2 a c\left(d+e x^2\right)\right)\right)+$$

$$i\left(-b+\sqrt{b^2-4 a c}\right)\left(A c\left(b d-2 a e\right)+a B\left(-2 c d+b e\right)\right)$$

$$\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{\frac{2 b-2\sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]-$$

$$i\left(A c\left(-b^2 d+4 a c d+b\sqrt{b^2-4 a c}\, d-2 a\sqrt{b^2-4 a c}\, e\right)+\right.$$

$$\left.a B\left(b\left(-b+\sqrt{b^2-4 a c}\right) e+c\left(-2\sqrt{b^2-4 a c}\, d+4 a e\right)\right)\right)\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}$$

$$\left.\sqrt{\frac{2 b-2\sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)$$

## Problem 27: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{\left(a + b x^2 + c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 398 leaves, 4 steps):

$$\frac{x \left(A b^2 - a b B - 2 a A c + (A b - 2 a B) c x^2\right)}{a \left(b^2 - 4 a c\right) \sqrt{a + b x^2 + c x^4}} - \frac{(A b - 2 a B) \sqrt{c}\, x \sqrt{a + b x^2 + c x^4}}{a \left(b^2 - 4 a c\right) \left(\sqrt{a} + \sqrt{c}\, x^2\right)} + \left((A b - 2 a B) c^{1/4} \left(\sqrt{a} + \sqrt{c}\, x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c}\, x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(a^{3/4} \left(b^2 - 4 a c\right) \sqrt{a + b x^2 + c x^4}\right) + \left(\left(\sqrt{a}\, B - A \sqrt{c}\right) \left(\sqrt{a} + \sqrt{c}\, x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c}\, x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(2 a^{3/4} \left(b - 2 \sqrt{a} \sqrt{c}\right) c^{1/4} \sqrt{a + b x^2 + c x^4}\right)$$

Result (type 4, 497 leaves):

$$-\frac{1}{4\,a\,\left(b^2-4\,a\,c\right)\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(4\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\,\left(a\,B\,\left(b+2\,c\,x^2\right)-A\,\left(b^2-2\,a\,c+b\,c\,x^2\right)\right)+\right.$$

$$\mathrm{i}\,\left(A\,b-2\,a\,B\right)\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$

$$\mathrm{i}\,\left(-2\,a\,B\,\sqrt{b^2-4\,a\,c}+A\,\left(-b^2+4\,a\,c+b\,\sqrt{b^2-4\,a\,c}\right)\right)$$

$$\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

## Problem 28: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{A+B\,x^2}{\left(d+e\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 867 leaves, 9 steps) :

$$-\left(\left(x\left(a b c (B d-A e)-\left(b^2-2 a c\right)(A c d-A b e+a B e)+c\left(a B(2 c d-b e)-A\left(b c d-b^2 e+2 a c e\right)\right) x^2\right)\right)\Big/\left(a\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\sqrt{a+b x^2+c x^4}\right)\right)+$$

$$\frac{\sqrt{c}\left(a B(2 c d-b e)-A\left(b c d-b^2 e+2 a c e\right)\right) x \sqrt{a+b x^2+c x^4}}{a\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-$$

$$\frac{e^{3/2}(B d-A e) \operatorname{ArcTan}\left[\frac{\sqrt{c d^2-b d e+a e^2}\, x}{\sqrt{d}\sqrt{e}\sqrt{a+b x^2+c x^4}}\right]}{2\sqrt{d}\left(c d^2-b d e+a e^2\right)^{3/2}}-$$

$$\left(c^{1/4}\left(a B(2 c d-b e)-A\left(b c d-b^2 e+2 a c e\right)\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(a^{3/4}\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\sqrt{a+b x^2+c x^4}\right)+$$

$$\left(\left(\sqrt{a}\, B-A\sqrt{c}\right) c^{1/4}\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right]\right)\Big/$$

$$\left(2 a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(\sqrt{c}\, d-\sqrt{a}\, e\right)\sqrt{a+b x^2+c x^4}\right)+$$

$$\left(a^{3/4} e\left(\frac{\sqrt{c}\, d}{\sqrt{a}}+e\right)^2(B d-A e)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}} \operatorname{EllipticPi}\left[-\frac{\left(\sqrt{c}\, d-\sqrt{a}\, e\right)^2}{4\sqrt{a}\sqrt{c}\, d e}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(4 c^{1/4} d\left(c d^2-a e^2\right)\left(c d^2-b d e+a e^2\right)\sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 3361 leaves):

$$\left(-A b^2 c d x+a b B c d x+2 a A c^2 d x+A b^3 e x-a b^2 B e x-3 a A b c e x+2 a^2 B c e x-A b c^2 d x^3+2 a B c^2 d x^3+A b^2 c e x^3-a b B c e x^3-2 a A c^2 e x^3\right)\Big/$$

$$\left(a\left(-b^2+4ac\right)\left(c d^2-b d e+a e^2\right)\sqrt{a+b x^2+c x^4}\right)-\frac{1}{a\left(-b^2+4ac\right)\left(c d^2-b d e+a e^2\right)}$$

$$\left(-\left(\left(\mathrm{i}\,A\,b\,c\left(-b+\sqrt{b^2-4ac}\right)d\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.\right.\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\Big/$$

$$\left.\left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+b x^2+c x^4}\right)\right)+$$

$$\left(\mathrm{i}\,a\,B\,c\left(-b+\sqrt{b^2-4ac}\right)d\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+b x^2+c x^4}\right)+$$

$$\left(\mathrm{i}\,A\,b^2\left(-b+\sqrt{b^2-4ac}\right)e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+b x^2+c x^4}\right)-$$

$$\left(\mathrm{i}\,a\,b\,B\left(-b+\sqrt{b^2-4ac}\right)e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\mathrm{i}\,a\,A\,c\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\mathrm{i}\,a\,b\,B\,c\,d\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(\mathrm{i}\,\sqrt{2}\,a\,A\,c^2\,d\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\mathrm{i}\,a\,A\,b\,c\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+$$

$$\Bigg(i\sqrt{2}\,a^2 B c e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+$$

$$\Bigg(i\,a b^2 B e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\;\text{EllipticPi}\Bigg[$$

$$-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Bigg]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\Bigg(2\,i\sqrt{2}\,a^2 B c e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}$$

$$\text{EllipticPi}\Bigg[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],$$

$$\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Bigg]\Bigg)\Bigg/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\Bigg(i\,a A b^2 e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\;\text{EllipticPi}\Bigg[$$

$$-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Bigg]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,d\sqrt{a+bx^2+cx^4}\right)+$$

$$\left(2\,i\,\sqrt{2}\,a^2\,A\,c\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$
$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\,i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\right.$$
$$\left.\left.\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,d\,\sqrt{a+b\,x^2+c\,x^4}\right)\Bigg)$$

## Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{A+B\,x^2}{\left(d+e\,x^2\right)^2\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 1301 leaves, 15 steps):

$$\left(x\,\left(a\,b\,c\,\left(A\,e\,\left(2\,c\,d-b\,e\right)-B\,\left(c\,d^2-a\,e^2\right)\right)+\left(b^2-2\,a\,c\right)\,\left(a\,B\,e\,\left(2\,c\,d-b\,e\right)+A\,\left(c^2\,d^2+b^2\,e^2-c\,e\,\left(2\,b\,d+a\,e\right)\right)\right)-c\,\left(a\,B\,\left(2\,c^2\,d^2+b^2\,e^2-2\,c\,e\,\left(b\,d+a\,e\right)\right)+A\,\left(2\,b^2\,c\,d\,e-4\,a\,c^2\,d\,e-b^3\,e^2-b\,c\,\left(c\,d^2-3\,a\,e^2\right)\right)\right)\,x^2\right)\right)\Big/\left(a\,\left(b^2-4\,a\,c\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$
$$\left(\sqrt{c}\,\left(a\,B\,d\,\left(-4\,c^2\,d^2-3\,b^2\,e^2+4\,c\,e\,\left(b\,d+2\,a\,e\right)\right)+A\,\left(2\,b^3\,d\,e^2+2\,b\,c\,d\,\left(c\,d^2-3\,a\,e^2\right)-4\,a\,c\,e\,\left(-2\,c\,d^2+a\,e^2\right)+b^2\,\left(-4\,c\,d^2\,e+a\,e^3\right)\right)\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}\right)\Big/\left(2\,a\,\left(-b^2+4\,a\,c\right)\,d\,\left(c\,d^2+e\,\left(-b\,d+a\,e\right)\right)^2\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\right)-$$
$$\frac{e^3\,\left(B\,d-A\,e\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{2\,d\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\left(d+e\,x^2\right)}+$$
$$\left(e^{3/2}\,\left(A\,e\,\left(7\,c\,d^2-e\,\left(4\,b\,d-a\,e\right)\right)-B\,d\,\left(5\,c\,d^2-e\,\left(2\,b\,d+a\,e\right)\right)\right)\right.$$
$$\left.\text{ArcTan}\left[\frac{\sqrt{c\,d^2-b\,d\,e+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+b\,x^2+c\,x^4}}\right]\right)\Big/\left(4\,d^{3/2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^{5/2}\right)-$$
$$\left(c^{1/4}\,\left(a\,B\,d\,\left(4\,c^2\,d^2+3\,b^2\,e^2-4\,c\,e\,\left(b\,d+2\,a\,e\right)\right)-A\,\left(2\,b^3\,d\,e^2+2\,b\,c\,d\,\left(c\,d^2-3\,a\,e^2\right)-4\,a\,c\,e\,\left(-2\,c\,d^2+a\,e^2\right)+b^2\,\left(-4\,c\,d^2\,e+a\,e^3\right)\right)\right)\right.$$
$$\left.\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}-\frac{b}{4\,\sqrt{a}\,\sqrt{c}}\right]\right)\Big/$$
$$\left(2\,a^{3/4}\,\left(b^2-4\,a\,c\right)\,d\,\left(c\,d^2+e\,\left(-b\,d+a\,e\right)\right)^2\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(c^{1/4}\left(a\sqrt{c}\,e\,(B d-2 A e)+\sqrt{a}\,(B d-A e)\,(c d-b e)+A\sqrt{c}\,d\,(-c d+b e)\right)\right.$$

$$\left.\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right]\right)\bigg/$$

$$\left(2 a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)d\left(-\sqrt{c}\,d+\sqrt{a}\,e\right)\left(-c d^2+e\,(b d-a e)\right)\sqrt{a+b x^2+c x^4}\right)-$$

$$\left(e\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\left(A e\left(7 c d^2-e\,(4 b d-a e)\right)-B d\left(5 c d^2-e\,(2 b d+a e)\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\bigg/$$

$$\left(8 a^{1/4}c^{1/4}d^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(c d^2-b d e+a e^2\right)^2\sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 8031 leaves):

$$\sqrt{a+b x^2+c x^4}$$

$$\left(-\frac{e^3\,(B d-A e)\,x}{2 d\left(c d^2-b d e+a e^2\right)^2\left(d+e x^2\right)}+\frac{1}{a\left(-b^2+4 a c\right)\left(c d^2-b d e+a e^2\right)^2\left(a+b x^2+c x^4\right)}\right.$$

$$\left(-A b^2 c^2 d^2 x+a b B c^2 d^2 x+2 a A c^3 d^2 x+2 A b^3 c d e x-2 a b^2 B c d e x-6 a A b c^2 d e x+\right.$$
$$4 a^2 B c^2 d e x-A b^4 e^2 x+a b^3 B e^2 x+4 a A b^2 c e^2 x-3 a^2 b B c e^2 x-2 a^2 A c^2 e^2 x-$$
$$A b c^3 d^2 x^3+2 a B c^3 d^2 x^3+2 A b^2 c^2 d e x^3-2 a b B c^2 d e x^3-4 a A c^3 d e x^3-A b^3 c e^2 x^3+$$
$$\left.a b^2 B c e^2 x^3+3 a A b c^2 e^2 x^3-2 a^2 B c^2 e^2 x^3\right)-\frac{1}{2 a\left(-b^2+4 a c\right)d\left(c d^2-b d e+a e^2\right)^2}$$

$$\left(-\left(\left(\left(\mathbb{i}\,A b c^2\left(-b+\sqrt{b^2-4 a c}\right)d^3\sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.\right.\right.\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,x\right],\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\,x\right],\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right/$$

$$\left.\left.\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\sqrt{a+b x^2+c x^4}\right)\right)\right)+$$

$$\left(\mathbb{i}\sqrt{2}\,a B c^2\left(-b+\sqrt{b^2-4 a c}\right)d^3\sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(\mathrm{i}\,\sqrt{2}\,A\,b^2\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\mathrm{i}\,\sqrt{2}\,a\,b\,B\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(2\,\mathrm{i}\,\sqrt{2}\,a\,A\,c^2\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{a+b x^2+c x^4}\right)-$$

$$\left(i A b^3 \left(-b+\sqrt{b^2-4 a c}\right) d e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{a+b x^2+c x^4}\right)+$$

$$\left(3 i a b^2 B \left(-b+\sqrt{b^2-4 a c}\right) d e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\Big/$$

$$\left(2 \sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{a+b x^2+c x^4}\right)+$$

$$\left(3 i a A b c \left(-b+\sqrt{b^2-4 a c}\right) d e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{a+b x^2+c x^4}\right)-$$

$$\left(2 i \sqrt{2}\ a^2 B c \left(-b+\sqrt{b^2-4 a c}\right) d e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(i\,a\,A\,b^2\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(i\,\sqrt{2}\,a^2\,A\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(i\,\sqrt{2}\,a\,b\,B\,c^2\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(2\,\mathrm{i}\,\sqrt{2}\,a\,A\,c^3\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\mathrm{i}\,a\,b^2\,B\,c\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(2\,\mathrm{i}\,\sqrt{2}\,a\,A\,b\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(6\,\mathrm{i}\,\sqrt{2}\,a^2\,B\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(3\,\mathrm{i}\,a\,A\,b^2\,c\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\left(i\sqrt{2}\,a^2 b B c d e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\left(4\,i\sqrt{2}\,a^2 A c^2 d e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+$$

$$\left(5\,i\,a b^2 B c d^2 e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\left(10\,i\sqrt{2}\,a^2 B c^2 d^2 e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\right.$$

$$\left.\left.\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\left(i\sqrt{2}\,a b^3 B d e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\ \mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\ \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(7\,\mathrm{i}\,a\,A\,b^2\,c\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\ \mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\ \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(4\,\mathrm{i}\,\sqrt{2}\,a^2\,b\,B\,c\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\ \mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\ \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(14\,\mathrm{i}\,\sqrt{2}\,a^2\,A\,c^2\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\ \mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\ \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(2\,\mathrm{i}\,\sqrt{2}\,a\,A\,b^3\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\ \mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\ \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(i\, a^2 b^2 B e^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4 a c}\right) e}{2 c d}, i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\left(8 i \sqrt{2}\, a^2 A b c e^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4 a c}\right) e}{2 c d}, i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) +$$

$$\left(2 i \sqrt{2}\, a^3 B c e^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4 a c}\right) e}{2 c d}, i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\left(i\, a^2 A b^2 e^4 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4 a c}\right) e}{2 c d}, i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ d \sqrt{a+b x^2+c x^4}\right) +$$

$$\left(2 i \sqrt{2}\, a^3 A c e^4 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4 a c}\right) e}{2 c d}, i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right],\right.\right.$$

$$\left.\left.\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Bigg/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\; d \sqrt{a+b x^2+c x^4}\right)\Bigg)$$

## Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a}+\sqrt{c}\, x^2}{\left(d+e x^2\right) \sqrt{a+b x^2+c x^4}}\, dx$$

Optimal (type 4, 273 leaves, 1 step):

$$-\frac{\left(\sqrt{c}\, d-\sqrt{a}\, e\right) \text{ArcTan}\left[\frac{\sqrt{c d^2-b d e+a e^2}\, x}{\sqrt{d} \sqrt{e} \sqrt{a+b x^2+c x^4}}\right]}{2 \sqrt{d} \sqrt{e} \sqrt{c d^2-b d e+a e^2}}+$$

$$\left(\left(\sqrt{c}\, d+\sqrt{a}\, e\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\; \text{EllipticPi}\left[-\frac{\left(\sqrt{c}\, d-\sqrt{a}\, e\right)^2}{4 \sqrt{a} \sqrt{c}\, d e},\right.\right.$$

$$\left.\left.2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Bigg/ \left(4 a^{1/4} c^{1/4} d e \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 310 leaves):

$$-\left(\left(i \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\right.\right.$$

$$\left(\sqrt{c}\, d\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+\right.$$

$$\left(-\sqrt{c}\, d+\sqrt{a}\, e\right) \text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4 a c}\right) e}{2 c d}, i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\right.$$

$$\left.\left.\left.\left.\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)\right) \Bigg/ \left(\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\; d e \sqrt{a+b x^2+c x^4}\right)\right)$$

## Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{\frac{c}{a}}\, x^2}{\left(d+e x^2\right) \sqrt{a+b x^2+c x^4}}\, dx$$

Optimal (type 4, 271 leaves, 1 step):

$$-\frac{\left(\sqrt{\frac{c}{a}}\,d-e\right)\text{ArcTan}\left[\frac{\sqrt{c\,d^2-b\,d\,e+a\,e^2}\;x}{\sqrt{d}\;\sqrt{e}\;\sqrt{a+b\,x^2+c\,x^4}}\right]}{2\,\sqrt{d}\;\sqrt{e}\;\sqrt{c\,d^2-b\,d\,e+a\,e^2}}+$$

$$\left(\left(\sqrt{\frac{c}{a}}\,d+e\right)\left(1+\sqrt{\frac{c}{a}}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{a\left(1+\sqrt{\frac{c}{a}}\,x^2\right)^2}}\;\text{EllipticPi}\left[-\frac{\left(\sqrt{\frac{c}{a}}\,d-e\right)^2}{4\,\sqrt{\frac{c}{a}}\,d\,e},\right.\right.$$

$$\left.\left.2\,\text{ArcTan}\left[\left(\frac{c}{a}\right)^{1/4}x\right],\frac{1}{4}\left(2-\frac{b\,\sqrt{\frac{c}{a}}}{c}\right)\right]\right)\Big/\left(4\left(\frac{c}{a}\right)^{1/4}d\,e\,\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 312 leaves):

$$-\left(\left(\mathrm{i}\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\;\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\right.\right.$$

$$\left(\sqrt{\frac{c}{a}}\,d\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$\left(-\sqrt{\frac{c}{a}}\,d+e\right)\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\right.$$

$$\left.\left.\left.\left.\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,d\,e\,\sqrt{a+b\,x^2+c\,x^4}\right)\right)$$

## Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{946+315\,x^2}{\left(7+5\,x^2\right)\sqrt{2+3\,x^2+x^4}}\,dx$$

Optimal (type 4, 106 leaves, 4 steps):

$$\frac{631\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\;\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{2\,\sqrt{2}\;\sqrt{2+3\,x^2+x^4}}-\frac{2525\left(2+x^2\right)\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{14\,\sqrt{2}\;\sqrt{\frac{2+x^2}{1+x^2}}\;\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 74 leaves):

$$-\frac{1}{7\sqrt{2+3x^2+x^4}}\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\left(441\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+505\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

## Problem 33: Unable to integrate problem.

$$\int\frac{(A+B x^2)\,(d+e x^2)^q}{a+b x^2+c x^4}\,dx$$

Optimal (type 6, 218 leaves, 6 steps):

$$\frac{1}{b-\sqrt{b^2-4ac}}\left(B-\frac{b B-2 A c}{\sqrt{b^2-4ac}}\right)x\,(d+e x^2)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2 c x^2}{b-\sqrt{b^2-4ac}},-\frac{e x^2}{d}\right]+\frac{1}{b+\sqrt{b^2-4ac}}\left(B+\frac{b B-2 A c}{\sqrt{b^2-4ac}}\right)x\,(d+e x^2)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4ac}},-\frac{e x^2}{d}\right]$$

Result (type 8, 33 leaves):

$$\int\frac{(A+B x^2)\,(d+e x^2)^q}{a+b x^2+c x^4}\,dx$$

## Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx$$

Optimal (type 3, 106 leaves, 11 steps):

$$-\frac{1}{2}\text{ArcTan}\left[\sqrt{3}-2\sqrt{1+x^2}\right]+\frac{1}{2}\text{ArcTan}\left[\sqrt{3}+2\sqrt{1+x^2}\right]+\frac{1}{4}\sqrt{3}\,\text{Log}\left[2+x^2-\sqrt{3}\,\sqrt{1+x^2}\right]-\frac{1}{4}\sqrt{3}\,\text{Log}\left[2+x^2+\sqrt{3}\,\sqrt{1+x^2}\right]$$

Result (type 3, 103 leaves):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{1+x^2}}{\sqrt{-1-i\sqrt{3}}}\right]}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{3}\right)}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{1+x^2}}{\sqrt{-1+i\sqrt{3}}}\right]}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{3}\right)}}$$

## Problem 35: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b x^2+c x^4}}{a d-c d x^4}\,dx$$

Optimal (type 3, 145 leaves, 4 steps):

$$-\frac{\sqrt{b-2\sqrt{a}\sqrt{c}}\ \text{ArcTanh}\left[\frac{\sqrt{b-2\sqrt{a}\sqrt{c}}\ x}{\sqrt{a+b x^2+c x^4}}\right]}{4\sqrt{a}\sqrt{c}\ d}+\frac{\sqrt{b+2\sqrt{a}\sqrt{c}}\ \text{ArcTanh}\left[\frac{\sqrt{b+2\sqrt{a}\sqrt{c}}\ x}{\sqrt{a+b x^2+c x^4}}\right]}{4\sqrt{a}\sqrt{c}\ d}$$

Result (type 4, 441 leaves):

$$\left(i\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\ \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\right.$$
$$\left(2\sqrt{a}\sqrt{c}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\ x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]-\right.$$
$$\left(b+2\sqrt{a}\sqrt{c}\right)\text{EllipticPi}\left[\frac{-b-\sqrt{b^2-4 a c}}{2\sqrt{a}\sqrt{c}},\right.$$
$$\left.i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\ x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+\left(b-2\sqrt{a}\sqrt{c}\right)$$
$$\left.\left.\text{EllipticPi}\left[\frac{b+\sqrt{b^2-4 a c}}{2\sqrt{a}\sqrt{c}},i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\ x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)\right)\Big/$$
$$\left(2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\ d\sqrt{a+b x^2+c x^4}\right)$$

## Problem 36: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+b x^2-c x^4}}{a d+c d x^4}\,dx$$

Optimal (type 3, 239 leaves, 1 step):

$$-\frac{\sqrt{b+\sqrt{b^2+4 a c}}\ \text{ArcTan}\left[\frac{\sqrt{b+\sqrt{b^2+4 a c}}\ x \left(b-\sqrt{b^2+4 a c}-2 c x^2\right)}{2 \sqrt{2} \sqrt{a} \sqrt{c} \sqrt{a+b x^2-c x^4}}\right]}{2 \sqrt{2} \sqrt{a} \sqrt{c}\ d}+$$

$$\frac{\sqrt{-b+\sqrt{b^2+4 a c}}\ \text{ArcTanh}\left[\frac{\sqrt{-b+\sqrt{b^2+4 a c}}\ x \left(b+\sqrt{b^2+4 a c}-2 c x^2\right)}{2 \sqrt{2} \sqrt{a} \sqrt{c} \sqrt{a+b x^2-c x^4}}\right]}{2 \sqrt{2} \sqrt{a} \sqrt{c}\ d}$$

Result (type 4, 432 leaves):

$$\frac{1}{4 \sqrt{a} \sqrt{c} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\ d \sqrt{a+b x^2-c x^4}} \sqrt{2+\frac{4 c x^2}{-b+\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}$$

$$\left(2 \mathrm{i} \sqrt{a} \sqrt{c}\ \text{EllipticF}\left[\mathrm{i}\ \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\ x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]+\right.$$

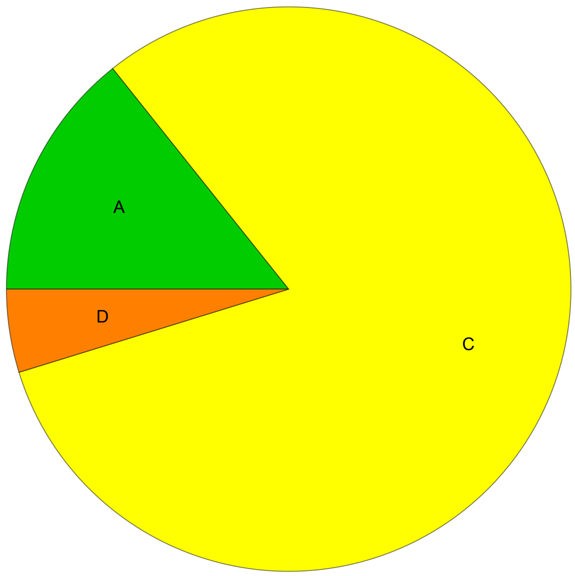
$$\left(b-2 \mathrm{i} \sqrt{a} \sqrt{c}\right) \text{EllipticPi}\left[-\frac{\mathrm{i} \left(b+\sqrt{b^2+4 a c}\right)}{2 \sqrt{a} \sqrt{c}},\right.$$

$$\left.\mathrm{i}\ \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\ x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]-\left(b+2 \mathrm{i} \sqrt{a} \sqrt{c}\right)$$

$$\left.\text{EllipticPi}\left[\frac{\mathrm{i} \left(b+\sqrt{b^2+4 a c}\right)}{2 \sqrt{a} \sqrt{c}}, \mathrm{i}\ \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\ x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right)$$

# Summary of Integration Test Results

42 integration problems




A - 6 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 34 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts